UNIVERSITY OF CAMBRIDGE

Machine Learning and the Physical World

Lecture 1 : Introduction

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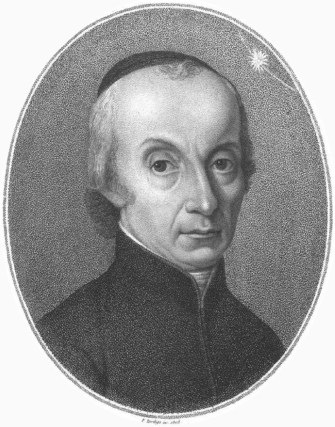
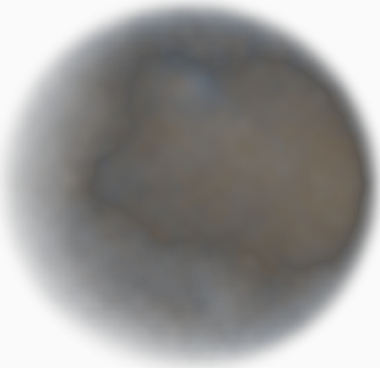
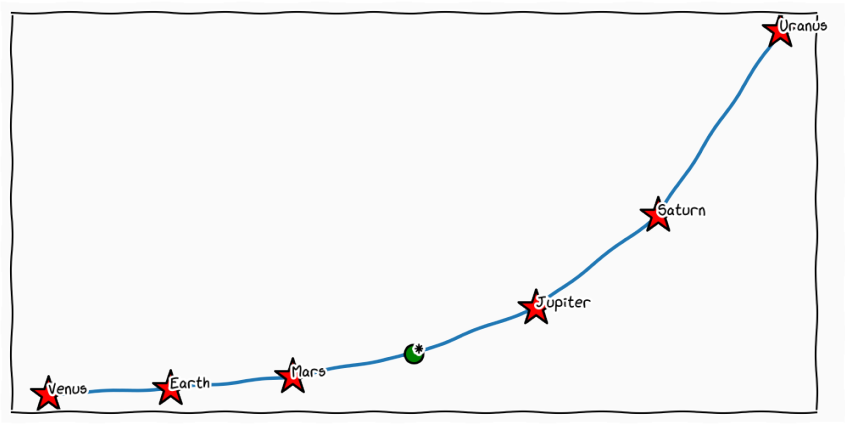
5th of October, 2023

http://carlhenrik.com



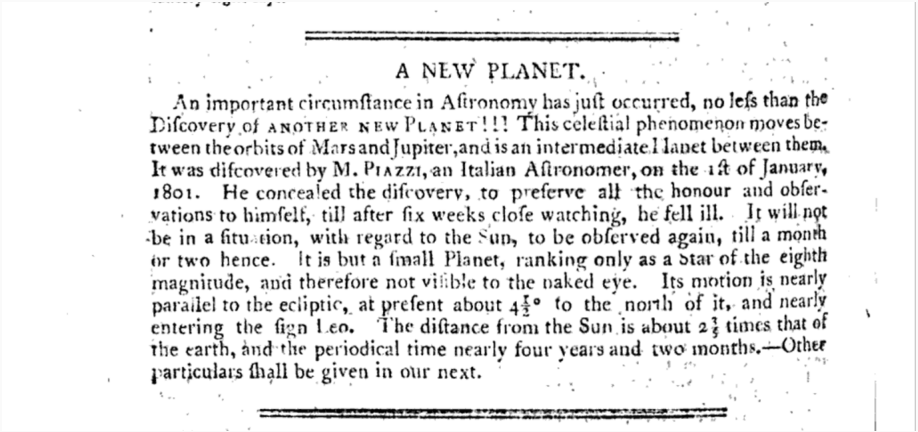


Titus-Bode "Law"



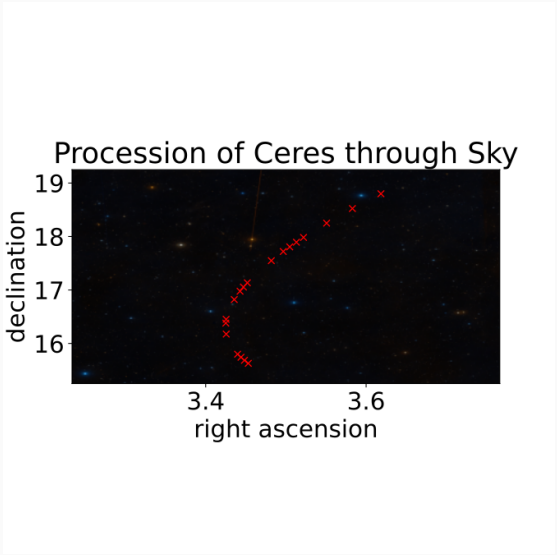
Beobachtungen des zu Palermo d. 1. Jan. 1801 von Prof. *Piazzi* neu entdeckten Gestirns.

1801		Mittlere Sonnen-Zeit		Gerade Aufsteig. in Zeit		Gerade Aufsteigung in Graden		Nördl. Abweich.		Geocentrische Länge		Geocentr. Breite		Ort der Sonne + 20″ Aberration		Logar. d. Distanz ☉ ♁
		St ′ ″		St ′ ″		° ′ ″		° ′ ″		♑ ° ′ ″		° ′ ″		♑ ° ′ ″		
Jan.	1	8 43	27,8	3 27	11, 25	51 47	48, 8	15 37	45, 5	1 23	22 58, 2	3 6	42, 1	9 11	1 30, 9	9, 9926156
	2	8 39	4,6	3 26	53, 85	51 43	27, 8	15 41	5, 5	1 23	19 44, 3	3 2	24, 9	9 12	2 28, 6	9, 9926317
	3	8 34	53,3	3 26	38, 41	51 39	36, 0	15 44	31, 6	1 23	16 58, 6	2 58	9, 9	9 13	3 26, 6	9, 9926324
	4	8 30	42,1	3 26	23, 16	51 35	47, 3	15 47	57, 6	1 23	14 15, 5	2 53	55, 6	9 14	4 24, 9	9, 9926418
	10	8 6	15,8	3 25	32, 11	51 23	1, 5	16 10	32, 0	1 23	7 59, 1	2 29	0, 6	9 20	10 17, 5	9, 9927641
	11	8 2	17,5	3 25	29, 73	51 22	26, 0	. . .	. . . .	. . .	. . . . .	. . .	. . . .	. . .	. . . . .	. . . .
	13	7 54	26,2	3 25	30, 30	51 22	34, 5	16 22	49, 5	1 23	10 27, 6	2 16	59, 7	9 23	12 13, 8	9, 9928490
	14	7 50	31,7	3 25	31, 72	51 22	55, 8	16 27	8, 7	1 23	12 1, 2	2 12	56, 7	9 24	14 13, 5	9, 9928809
	17	. . .	. . . .	. . .	. . . .	. . .	. . . .	16 40	13, 0	. . .	. . . . .	. . .	. . . .	. . .	. . . . .	. . . .
	18	7 35	11,3	3 25	55, 11	51 28	45, 0	. . .	. . . .	. . .	. . . . .	. . .	. . . .	. . .	. . . . .	. . . .
	19	7 31	28,5	3 26	8, 15	51 32	2, 3	16 49	16, 1	1 23	25 59, 2	1 53	38, 2	9 29	19 53, 8	9, 9930607
	21	7 24	2,7	3 26	34, 27	51 38	34, 1	16 58	35, 9	1 23	34 21, 3	1 46	6, 0	10 1	20 40, 3	9, 9931434
	22	7 20	21,7	3 26	49, 42	51 42	21, 3	17 3	18, 5	1 23	39 1, 8	1 42	28, 1	10 2	21 32, 0	9, 9931886
	23	7 16	43,5	3 27	6, 90	51 46	43, 5	17 8	5, 5	1 23	44 15, 7	1 38	52, 1	10 3	22 22, 7	9, 9932348
	28	6 58	51,3	3 28	54, 53	52 13	38, 3	17 32	54, 1	1 24	15 15, 7	1 21	6, 9	10 8	26 20, 1	9, 9935062
	30	6 51	52,9	3 29	48, 14	52 27	2, 1	17 43	11, 0	1 24	30 9, 0	1 14	16, 0	10 10	27 46, 2	9, 9936332
	31	6 48	26,4	3 30	17, 25	52 34	18, 8	17 48	21, 5	1 24	38 7, 3	1 10	54, 6	10 11	28 28, 5	9, 9937007
Febr.	1	6 44	59,9	3 30	47, 21	52 41	48, 0	17 53	36, 3	1 24	46 19, 3	1 7	30, 9	10 12	29 9, 6	9, 9937703
	2	6 41	35,8	3 31	19, 06	52 49	45, 9	17 58	57, 5	1 24	54 57, 9	1 4	1, 5	10 13	29 49, 9	9, 9938423
	5	6 31	31,5	3 33	2, 70	53 15	40, 5	18 15	1, 0	1 25	22 43, 4	0 54	28, 9	10 16	31 45, 5	9, 9940751
	8	6 21	39,2	3 34	58, 50	53 44	37, 5	18 31	23, 2	1 25	53 29, 5	0 45	5, 0	10 19	33 33, 3	9, 9943276
	11	6 11	58,2	3 37	6, 54	54 16	38, 1	18 47	58, 8	1 26	26 40, 0	0 36	2, 9	10 22	35 13, 4	9, 9945823

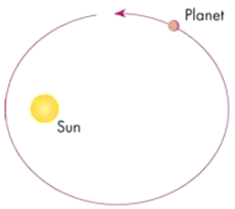


A NEW PLANET.

An important circumſtance in Aſtronomy has juſt occurred, no leſs than the Diſcovery of ANOTHER NEW PLANET!!! This celeſtial phenomenon moves between the orbits of Mars and Jupiter, and is an intermediate Planet between them. It was diſcovered by M. PIAZZI, an Italian Aſtronomer, on the 1ſt of January, 1801. He concealed the diſcovery, to preſerve all the honour and obſervations to himſelf, till after ſix weeks cloſe watching, he fell ill. It will not be in a ſituation, with regard to the Sun, to be obſerved again, till a month or two hence. It is but a ſmall Planet, ranking only as a Star of the eighth magnitude, and therefore not viſible to the naked eye. Its motion is nearly parallel to the ecliptic, at preſent about $4\frac{1}{2}°$ to the north of it, and nearly entering the ſign Leo. The diſtance from the Sun is about $2\frac{7}{8}$ times that of the earth, and the periodical time nearly four years and two months.—Other particulars ſhall be given in our next.

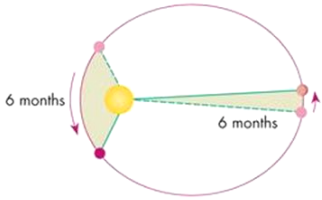

Procession of Ceres through Sky
declination
19
18
17
16
3.4
3.6
right ascension

Kepler's 3 Laws of Planetary Motion



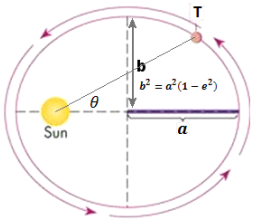
(1)

The orbits are ellipses



(2)

Equal areas in equal time



(3)

T = time to complete orbit

$T^2 \propto a^3$ a = semi-major axis

hier in der Nähe der Quadratur der Einfluss der Sonnen-Länge geringer ist, als in andern Lagen. Dr. *Gauss* glaubt daher, dass es nicht undienlich wäre, wenn man die Fehler der Sonnentafeln aus sehr genauen Beobachtungen für diese Zeiten bestimmte, und die Örter der Sonne hiernach verbesserte. Diese *vierten* Elemente sind nun folgende:

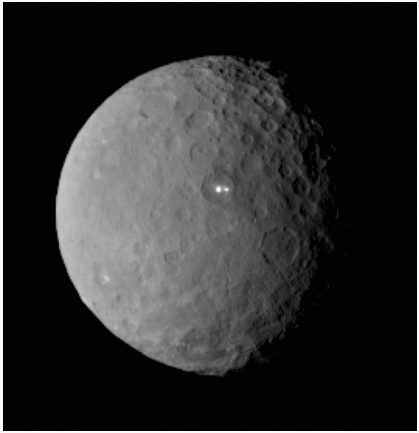
Sonnenferne . .	326°	27′	38″	Hieraus:	
☊	31	0	44	grösste Mittelp. Gleichung	9° 27′ 41″
Neigung . .	10	36	57		
Log. halb. gr. Axe	0,4420527			tägl. mittlere helioc. tropische Beweg.	770,″914
Excentricität . .	0,0825017				
Epoche 1800 31 Dec.	77°	36′	34″		

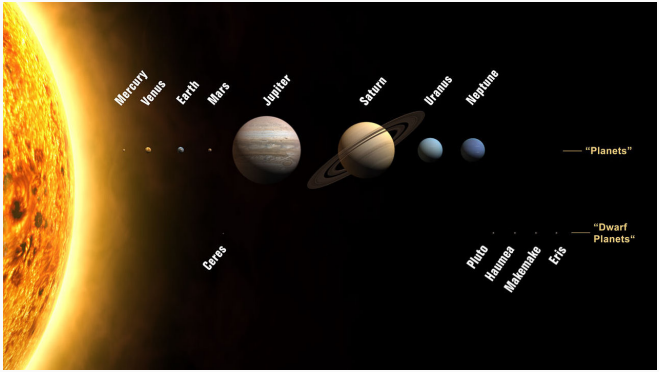
Aus diesen Elementen hat Dr. *Gauss* folgende Örter der *Ceres Ferdinandea* im voraus berechnet. Die Zeit ist mittlere für Mitternacht in *Palermo*.

1801	Geocentrische Länge		Geocentrische Breite nördl.		Logarith. des Abstandes von der ☉	Logarith. des Abstandes von der ☾	Verhältniss der gesehenen Helligk.
	Z	°	′	″			
Nov. 25	5	20	16	9 25	0,42181	0,40468	0,6102
Dec. 1	5	22	15	9 48	0,40940	0,40472	0,6459
7	5	24	7	10 12	0,39644	0,40479	0,6855
13	5	25	51	10 37	0,38296	0,40488	0,7290
19	5	27	27	11 4	0,36902	0,40499	0,7770
25	5	28	53	11 32	0,35468	0,40512	0,8295
31	6	0	10	12 1	0,34000	0,40528	0,8869

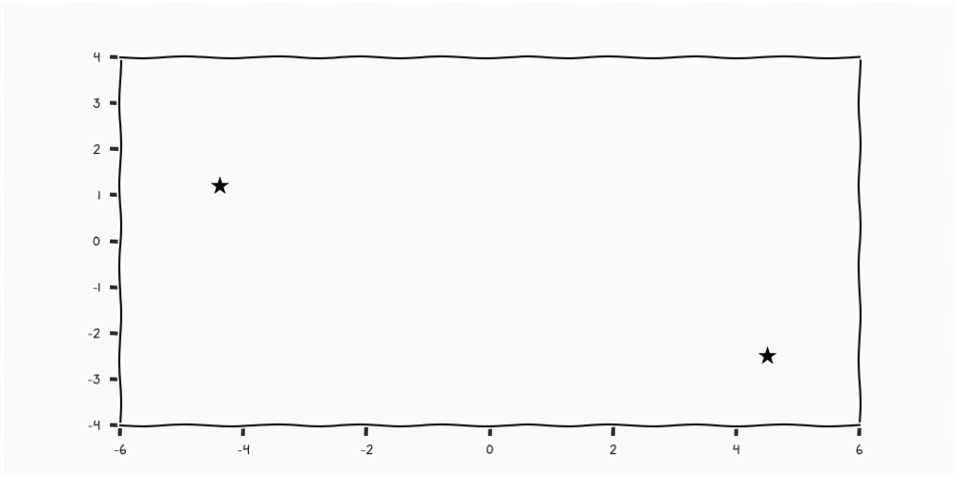
Sollte man den Ort des Planeten nach diesen Elementen genauer, oder auf eine längere Zeit berechnen wollen: so setzen wir zu diesem Behufe noch folgende Formeln hierher:

1) Zur

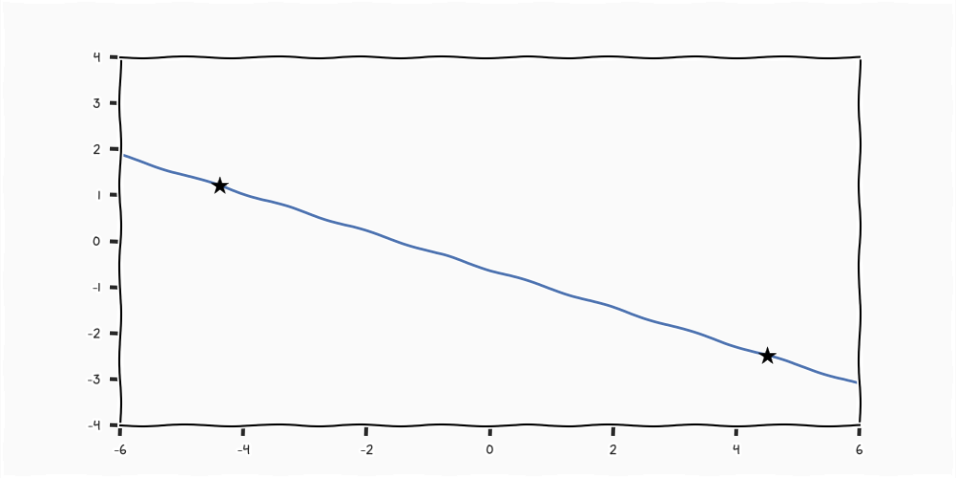



Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
"Planets"
"Dwarf Planets"
Ceres
Pluto
Haumea
Makemake
Eris

An Over-determined System



An Over-determined System



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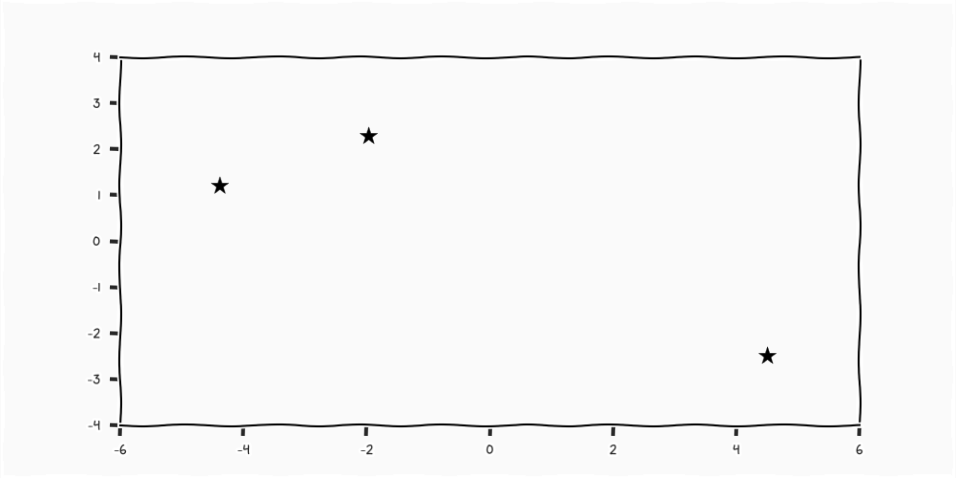

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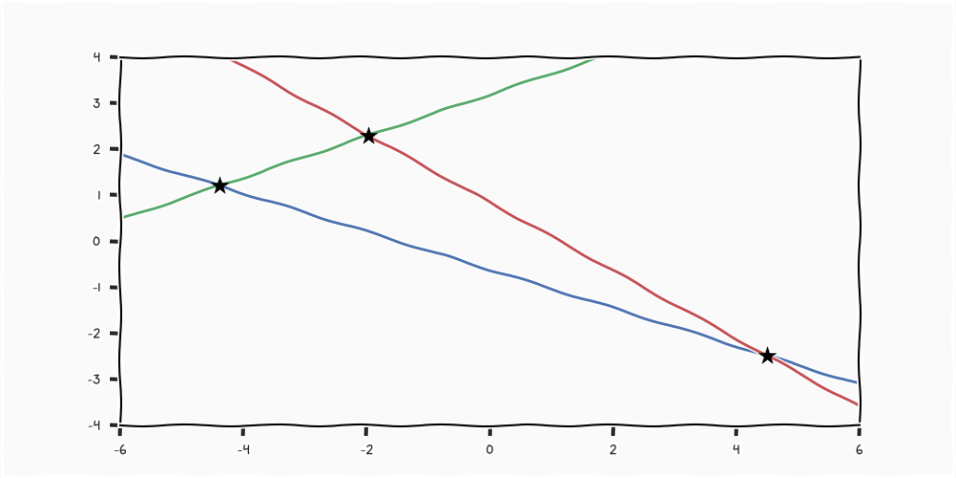
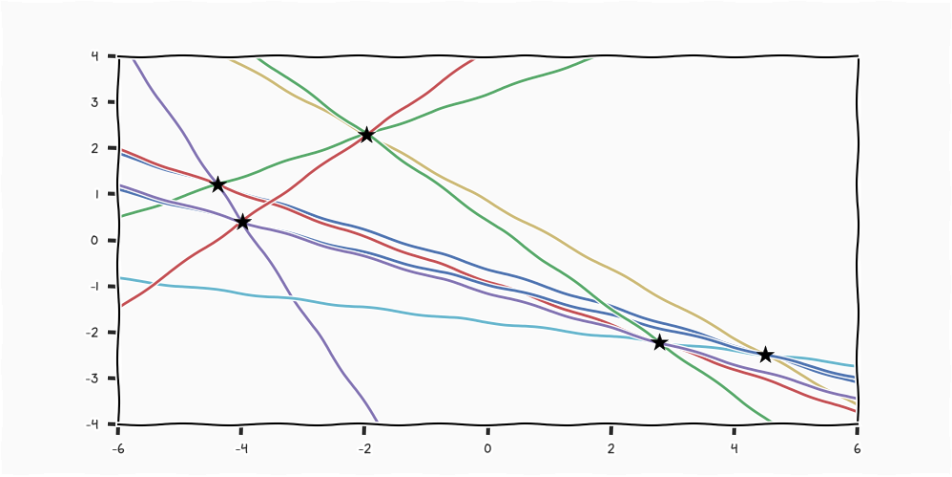
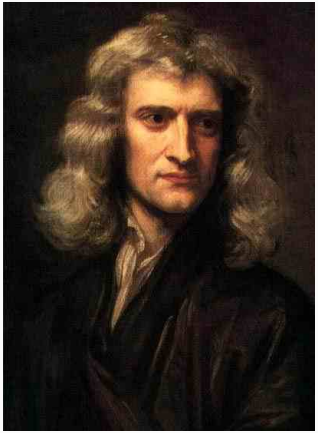


An Over-determined System





An Over-determined System

- An over-determined system means that we have more data than we need to determine our parameters.

An Over-determined System

- An over-determined system means that we have more data than we need to determine our parameters.
- The problem arises because the model is a simplification of the world and the data is therefore inconsistent with our model.

Laplace Demon Laplace, 1814

We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Does God Play Dice?[1]

This led to the idea of scientific determinism, which seems first to have been publicly expressed by the French scientist, Laplace.

– Stephen Hawkins

[1] Does God Play Dice? - Stephen Hawkins

All these efforts in the search for truth tend to lead [the human mind] back continually to the vast intelligence which we have just mentioned, but from which it will always remain infinitely removed.

– Pierre Simon Laplace, A Philosophical Essay on Probabilities
Laplace, 1814

Laplace Gremlin

"The curve described by a simple molecule of air or vapor is regulated in a manner just as certain as the planetary orbits; the only difference between them is that which comes from our ignorance. Probability is relative, in part to this ignorance, in part to our knowledge. We know that of three or greater number of events a single one ought to occur; but nothing induces us to believe that one of them will occur rather than the others. In this state of indecision it is impossible for us to announce their occurrence with certainty. It is, however, probable that one of these events, chosen at will, will not occur because we see several cases equally possible which exclude its occurrence, while only a single one favors it."

An Over-determined Sysatem

- Over determined system

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

An Over-determined Sysatem

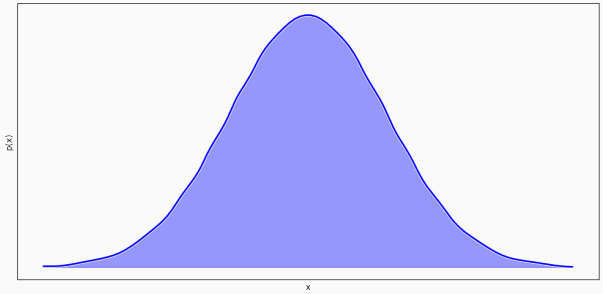
- Over determined system

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

- Parametrisation of ignorance

$$y_i = [a, b] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$

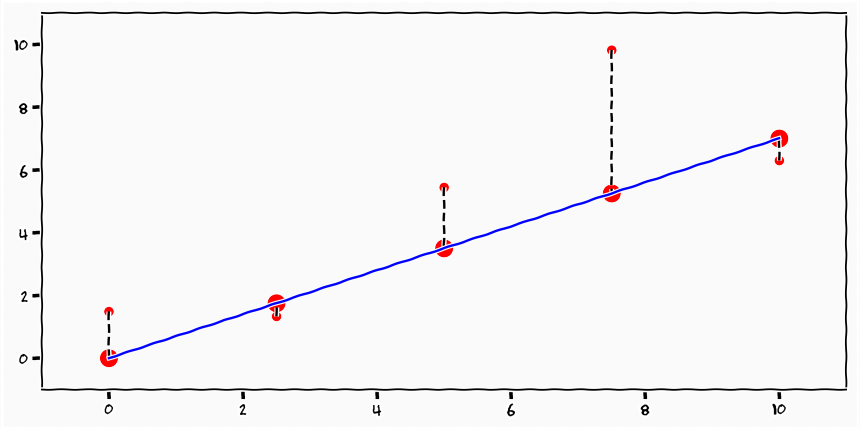
The Gaussian Distribution



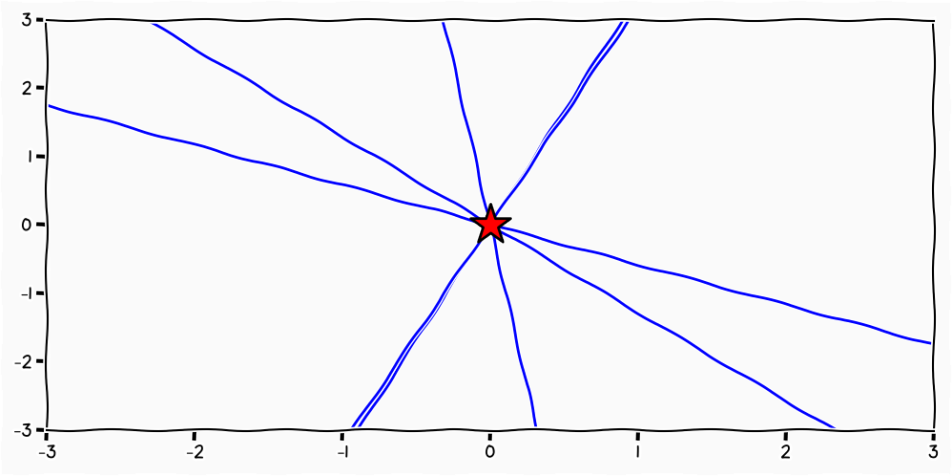
$$y_i = [a, b] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$

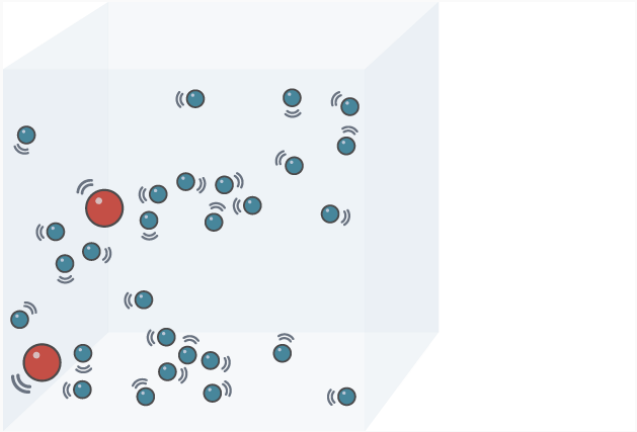
$$\epsilon_i \sim \mathcal{N}(0, \cdot)$$

Decomposition



An Under-determined System





Under-determined System

- Under determined system

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

Under-determined System

- Under determined system

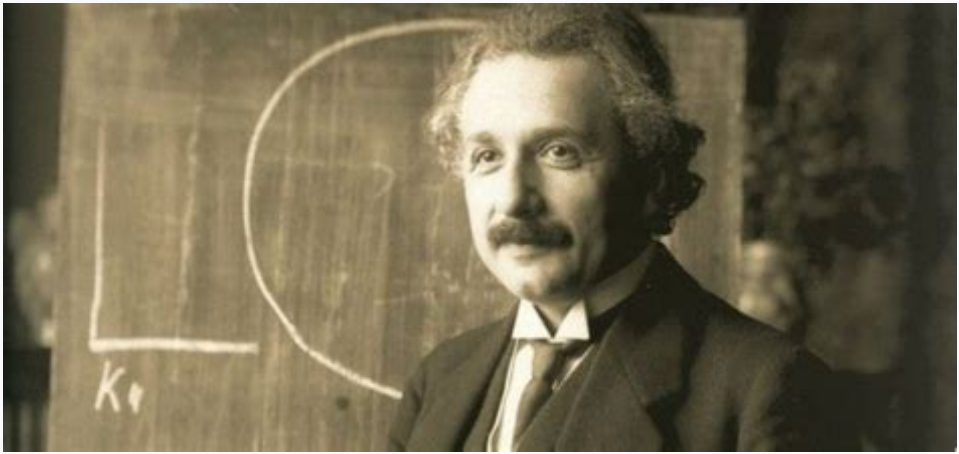
$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

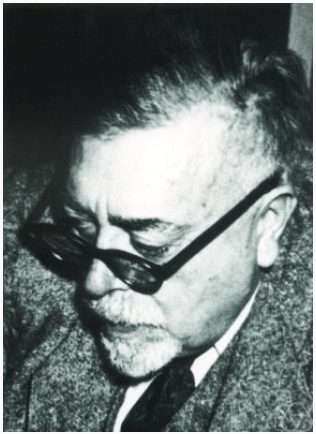
- Parametrisation of ignorance

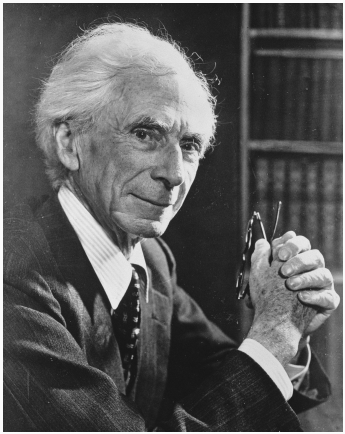
$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

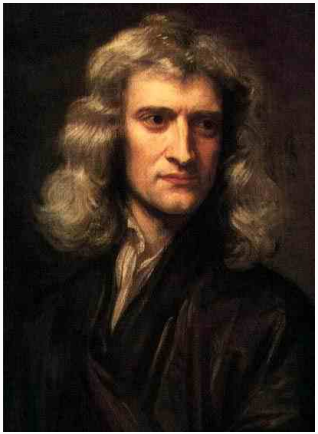
$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$$

Einstein

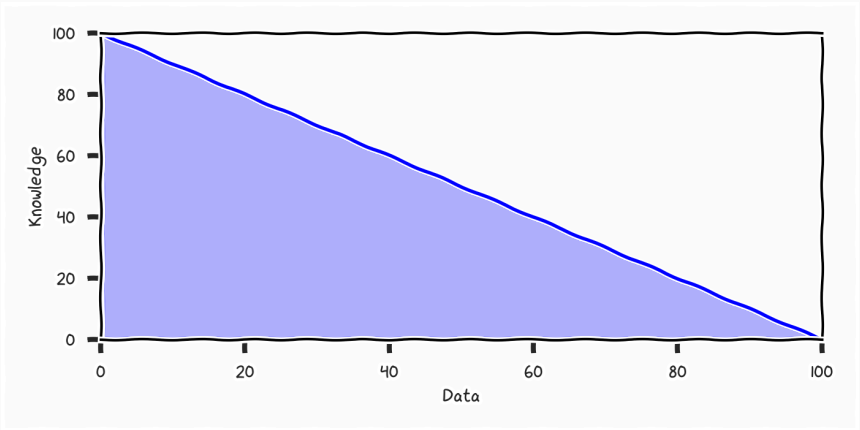


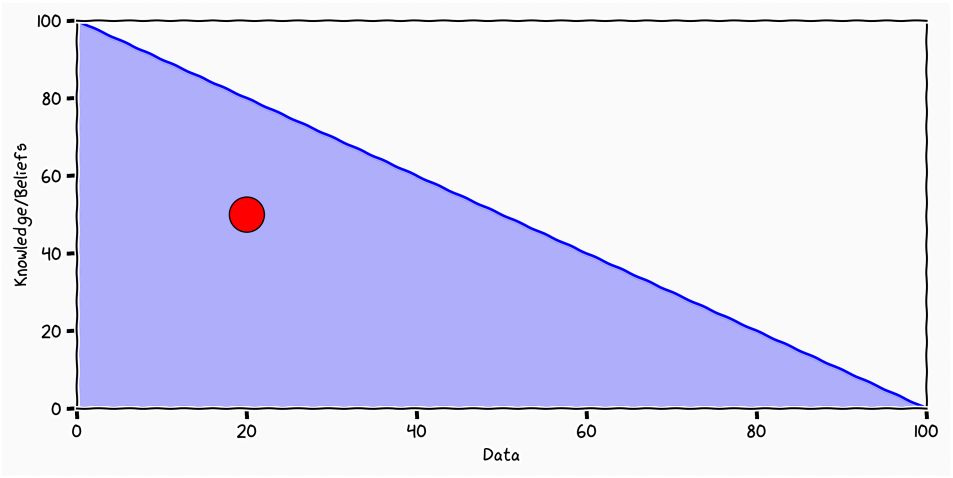




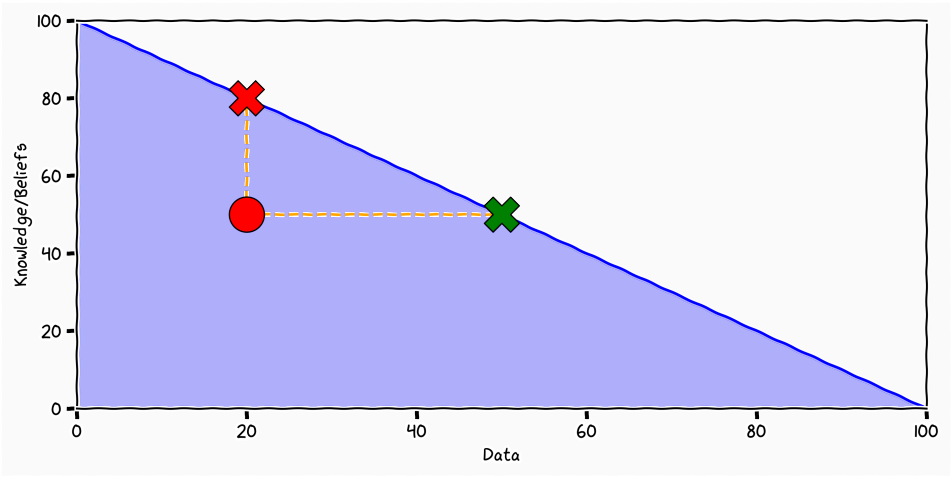


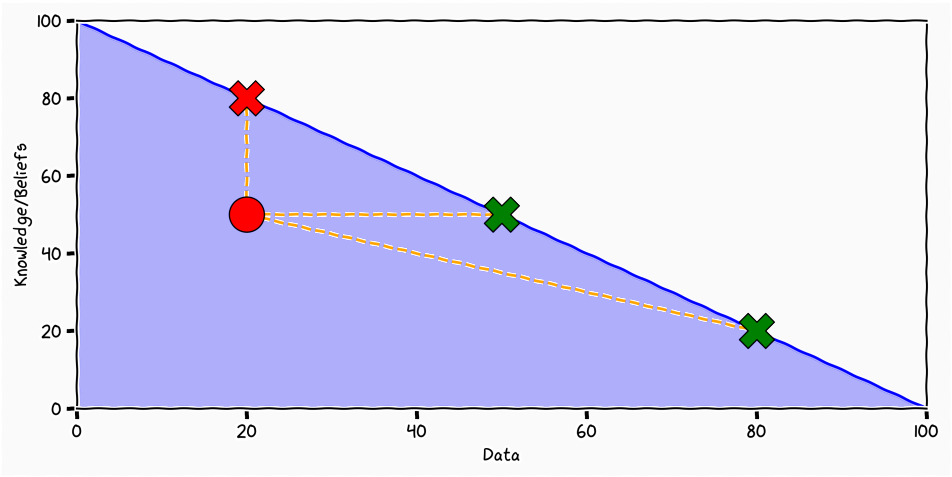
Machine Learning and the Physical World


100
80
60
40
20
0
Knowledge
0
20
40
60
80
100
Data



None


100
80
60
40
20
0
Knowledge/Beliefs
0
20
40
60
80
100
Data


100
80
60
40
20
0
Knowledge/Beliefs
0
20
40
60
80
100
Data

Aim of the course

We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

$$\text{Data} + \text{Model} \overset{\text{Compute}}{\rightarrow} \text{Prediction}$$

What is this course not

- Not about models

What is this course not

- Not about models
- Not about inference

What is this course not

- Not about models
- Not about inference
- Not about specific problems

Lectures

Week 1 Introduction (che29)

Week 1 Quantification of Beliefs (che29)

Week 2 Gaussian Processes (che29)

Week 2 Simulation (ndl21)

Week 3 Cancelled (TBC)

Week 3 Emulation (ndl21)

Week 4 Sequential Decision Making Under Uncertainty (che29)

Week 4 Probabilistic Numeric (che29)

Week 5 Experimental Design (ndl21)

Week 5 Multifidelity Modelling (ndl21)

Week 6 Project Introduction (che29)

Course website

https://mlatcl.github.io/mlphysical/

Week 6 Electrical Engines : Monumo - Markus Kaiser, Nicola Durrande

Guest Lectures

Week 6 Electrical Engines : Monumo - Markus Kaiser, Nicola Durrande

Week 7 Climate Models and Neural Processes : Prof. Rich Turner - Engineering

Guest Lectures

Week 6 Electrical Engines : Monumo - Markus Kaiser, Nicola Durrande

Week 7 Climate Models and Neural Processes : Prof. Rich Turner - Engineering

Week 7 Fluid Dynamics - Wittle Lab TBC

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Week 8 TBA

Assessment

- Indivudual Assessment ($2 \cdot 10\%$)

  Gaussian Processes deadline 20/10

  Sequential Decision Making deadline 3/11

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  Gaussian Processes deadline 20/10

  Sequential Decision Making deadline 3/11
- Group Assessment (80%)
  - pick your own simulation environment
  - deadline 18/1

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  - data
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